UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE	:	**MATH3801**
ASSESSMENT PATTERN	:	**MATH3801A**
MODULE NAME	:	**Logic**
DATE	:	**22-May-08**
TIME	:	**10:00**
TIME ALLOWED	:	**2 Hours 0 Minutes**

2007/08-MATH3801A-001-EXAM-31

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Give the definition of a propositional language L and the set of L-formulas.
   (b) Give the definition of the formation tree of an L-formula and give the unique formation tree for the L-formula

$$(((p \Rightarrow q)\&(q \vee r)) \Rightarrow (p \vee r)) \Rightarrow \neg(q \vee s)$$

   (c) State what it means for an L-formula to be in disjunctive normal form, and use the truth table method to put $(p \Rightarrow q) \Rightarrow r$ in disjunctive normal form.

2. (a) In the Predicate Calculus a language L is a set of predicates with each predicate P of L being n-ary for some $n \in \omega$. Define the variables, quantifiers, L-U-formulas for a set U, the degrees of formulas and L-U-sentences.
   (b) Let U be a nonempty set and S a set of unsigned L-U-sentences. Define what it means for S to be open and to be U-replete.
   (c) State Hintikka's Lemma and use it, along with the Tableau Method, to conclude that the sentence

$$(\forall x \forall y \forall z((Rxy \& Ryz) \Rightarrow Rxz))\&(\forall x \forall y(Rxy \Rightarrow \neg Ryx)\&(\forall x \exists y Rxy)$$

   is satisfiable in an infinite domain.

3. (a) Give the definition of primitive recursive function, partial recursive function and recursive function.
   (b) Give the definitions of Turing machine, Turing program and Turing computable partial function.
   (c) Disjoint subsets A and B of $\mathbb{N}$ are said to be recursively inseparable if there is no recursive set C such that $A \subset C$ and $C \cap B = \emptyset$. Show that $A = \{x : \phi_x(x) = 0\}$ and $B = \{x : \phi_x(x) = 1\}$ are disjoint recursively enumerable sets which are recursively inseparable.
   (d) For A as in part c), prove that $K \equiv_1 A$, where $K = \{x \in \mathbb{N} : \phi_x(x) \text{ converges}\}$.

4. (a) Let $K = \{x \in \omega : \phi_x(x) \text{ converges}\}$. Note that ϕ_x denotes the x^{th} partial recursive function. Show that K is recursively enumerable, but not recursive.

   (b) Show that the partial recursive functions are not closed under μ, that is, there is a partial recursive function ψ such that $\lambda x[\mu[\psi(x, y) = 0]]$ is not partial recursive.

   (c) Give an example of a partial recursive function defined on a recursively enumberable subset of ω that can not be extended to a total recursive function on ω.

5. (a) In the language of arithmetic with a countably infinite set of variables $x, y, z, \ldots$ consider the k-place partial function $\lambda x_1 \cdots x_k[\psi(x_1, \ldots, x_k)]$. Define what it means for this function to be arithmetically definable.

   (b) For each formula F in the language of arithmetic let $\sharp(F)$, be the Gödel number of F. Let $TrueSnt = \{\sharp S : S \text{ is a true sentence in the language of arithmetic}\}$. Show that every arithmetically definable set $A \subset \omega$ is reducible to $TrueSnt$.

   (c) Supposing that every partial recursive function is arithmetically definable, show that the set $K = \{x \in \omega : \phi_x(x) \text{ converges}\}$ is arithmetically definable.